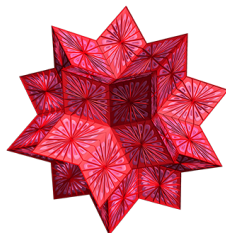Calculus I - Math 1210
Calculus I with Mathematica - Version 14©

Edit your document (remove extras and errors, ensure the rest works correctly) and turn-in your print-out. If needed, add hand written comments to your print-out.

1. This lab is a collection of problems whose concept is discussed in Calculus I course. Solve these problems using Mathematica commands. For help, see the Basic Mathematica Commands Lab, the Help menu of Mathematica, or your instructor.

Solve each of the following exercises. It might be desirable to do some work by hand. In any case, be sure to explain your work by typing in or writing out necessary comments.

A. Investigate the following limits. Decide if a limit exists by using a graph. If a limit exists, find its value. If a limit doesn't exist, explain why. Remember that the `Limit` command may give an incorrect answer.

(i) $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3}$

(ii) $\lim\limits_{x \to 1} \dfrac{(x - 1) \cos x}{|x - 1|}$

(iii) $\lim\limits_{x \to \infty} x^2 e^{-0.1x}$

B. Consider the curve defined by $x^2 \cos^2 y - \sin y = 0$.

(i) Find y' in terms of x and y.

(ii) Find y'' in terms of x, y and y'.

C. Consider the function $f(x) = 18x^4 - 8x^3 - 363x^2 + 840x + 20$.

(i) Find intervals in which it is decreasing and intervals in which it is increasing.

(ii) Find its local extreme points.

(iii) Find intervals in which its graph is concave downward and intervals in which its graph is concave upward.

(iv) Find inflection points of its graph.

(v) Graph this function. Show the results of the previous parts on the graph. (Hand write on the printout or type.)

D. Thelma is driving north on an old road, trying to find a telephone. The road becomes impassable at A, where a sign says that it is 2 miles to Highway Q (which Thelma knows goes east for 3 or 4 miles to Sleepy Hollow). She figures that she can hike to B at $4\,mph$ and jog to Sleepy Hollow at $5\,mph$. Find x to minimize her time. (Note: k is the unknown distance from C to Sleepy Hollow and $3 \le k \le 4$. Hint: Consider the sign of the first derivative of the time function; the 1st derivative test for absolute extremum.)
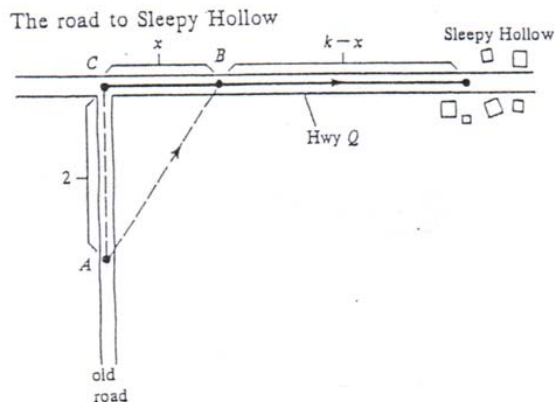
E. Evaluate the following integrals. If necessary, use hand evaluation.

(i) $\displaystyle\int_0^{\pi/2} x \sin 2x \, dx$

(ii) $\displaystyle\int \sqrt{\frac{x-1}{x^5}} \, dx$

(iii) $\displaystyle\int x^2(1 - x^3)^5 \, dx$ If the Mathematica's answer is too long, you don't have to print out the full solution, however, you should solve it by hand yourself.